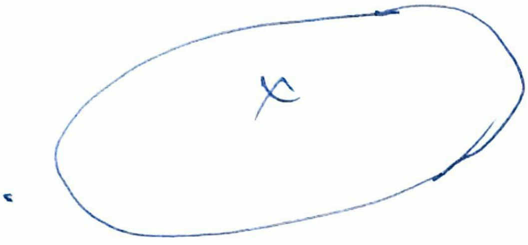
Date 31/07/2020

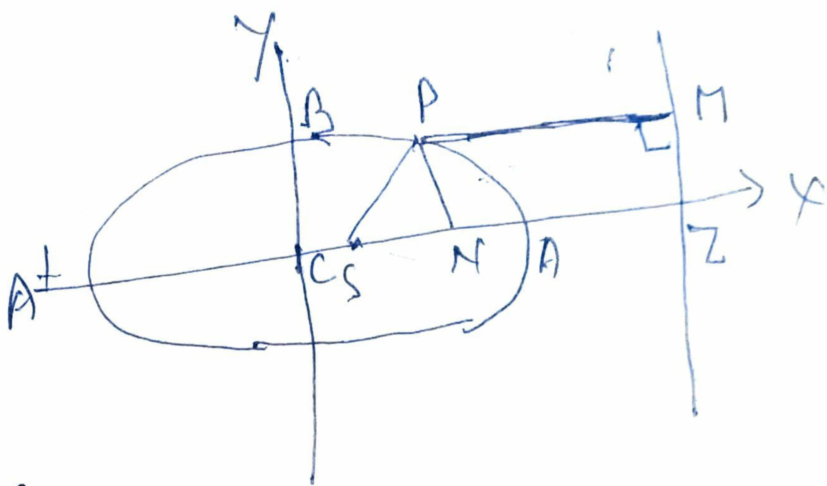
D1 - Mathematics

Topic - Equation of an ellipse

By Dr. R. A. Yadav, Deptt. of Maths, Marwari College, Darbhanga.

To find equation of an ellipse.





Let S be the focus, MZ be the directrix and $e\,(<1)$ be the eccentricity of the ellipse. Let SZ be perpendicular to MZ at Z. Let we divide SZ internally at A and externally at A' in the ratio $e:1$

Then $AS : AZ = e : 1$ $\quad \therefore AS = e AZ$ — (1)

therefore the point A' lies on the ellipse by definition

Similarly $A'S : A'Z = e : 1$

$\therefore A'S = e A'Z$ — (2)

Therefore point A' also lies on the ellipse

Let $AA' = 2a$ and let C be the middle point of AA' so that $CA = CA' = a$ — (3)

Now $AA' = AS + A'S = eAZ + eA'Z$

Let $P(x, y)$ be any point on the curve. Let PN and PM be perpendicular on the x-axis and the directrix respectively.

then $CN = x$, $PN = y$ — (6)

Now as P is on the ellipse $SP = eMP$

we have $SP^2 = e^2 MP^2 = e^2 ZN^2$ [$\because MP = ZN$]

But $SP^2 = SN^2 + PN^2$ [$\because \triangle SNP$ is right angled)

$$= (CN - CS)^2 + PN^2$$

$$= (x - ae)^2 + y^2 \quad (\text{by } (5) \& (6))$$

Also $ZN^2 = (CZ - CN)^2 = \left(\frac{a}{e} - x\right)^2$ by (4) & (6)

Hence by (7), (8) & (9)

$$(x - ae)^2 + y^2 = e^2 \left(\frac{a}{e} - x\right)^2$$

$$\Rightarrow x^2 - 2axe + a^2e^2 + y^2 = e^2 \left[\frac{a^2}{e^2} - 2\frac{a}{e}x + x^2\right)$$

$$\Rightarrow (1 - e^2)x^2 + y^2 = a^2(1 - e^2)$$

or dividing through by $(1 - e^2)$, we get

$$x^2 + \frac{y^2}{1 - e^2} = a^2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2(1 - e^2)} = \frac{a^2}{a^2} = 1 \quad — (10)$$

$\because CB = CB' = b$, co-ordinate of B are $(0, b)$.

thus $\frac{b^2}{a^2(1 - e^2)} = 1$ or $b^2 = a^2(1 - e^2)$ — (11)

eqn (10) becomes

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad — (12)$$

The eqn (12) is the equation of ellipse in standard form